

A BRIEF OVERVIEW OF THE CONTEMPORARY AND HYBRID APPLIED MATHEMATICAL METHODS FOR ELECTRICITY PRICES FORECASTING

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Abstract: *Since the creation of competitive and deregulated electricity markets two decades ago, electricity price forecasts (EPF) have gradually become one of the key mechanisms in the decision-making processes of energy companies and market participants, such as price-bidding strategy and planning for their generation facilities, sell and purchase contracts. The recent introduction of smart grids and the requirements for the integration of renewable energy sources has led to increased volatility and uncertainty in future prices that are formed on the basis of supply and demand. Now, more than ever, probabilistic forecasting of electricity prices (generation, demand and consumption) is an important tool for the planning and operation of energy systems. This article provides a brief overview of the trends in the development of mathematical methods for forecasting of electricity prices. The most up-to-date trends are covered in order to maximize the accuracy and reliability of forecasts by presenting the necessary guidance on the use of methods, measures and models in line with the paradigm of "maximizing the reliability of probability distributions subject to forecasts".*

Key words: *Electricity market, Electricity prices, Supply and demand, Renewable energy, Electricity price forecasting, Day-ahead market, Seasonality*

1. INTRODUCTION¹

Over the past decades, electricity price forecasting studies have been affected by the rapid process of liberalization and the dynamically changing features of global electricity markets, mainly influenced by the growing share of renewable energy sources and the ongoing evolution of technological progress and their further development. Both voluntary and regulatory, many energy sector institutions have contributed to continuously improving the transparency and quality of mostly freely available information and data on electricity prices and related time series. This, in turn, has helped researchers and practitioners to understand the pricing mechanisms which lead to a large number of articles focusing on electricity price forecasting.

In the period before 2005, there was only a small amount of published articles on this topic, while in the period between 2005 and 2012 the volume of published articles reached its first peak, followed by its current maximum. Research in electricity price forecasting originates from many different areas of science, as for example engineering, applied mathematics or statistics and computer science resulting with the application of a diverse and hybrid decision making technics containing a set of different approaches. However, most of these approaches have the common characteristic that they focus on forecasting electricity prices in the short term, in particular a day ahead with an hourly step, as described,

for example, in [21] or [20] and given in the literary review of forecasting electricity prices. By contrast, the methods of forecasting electricity prices that take longer to report over the medium and long term are rare and described in [22].

A large part of the articles published in this time horizon, which focus on forecasting electricity prices over the medium and long term, come from the fundamental models that capture the dynamics of systems, as for example the expected price functions for the market participants [23,24]. These types of models often do not use realistic time series of prices and related data, and therefore cannot provide a realistic time resolution of the price forecasts which typically occurs in the day-ahead markets. Nevertheless, there are models that are able to cover and predicts the electricity price hourly behavior as well as to provide medium to long-term forecasts.

1.1. The main goals of the report are to:

- Provide an overview of the modelling approaches for the EPF;
- Explain the complexity of the available solutions that the forecasting tools may offer or which might be encountered;
- Present a vision and suggest guidelines that should be taken into consideration over the next decade;

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- Emphasize the need for objective comparative studies of the EPF.

2. REVIEW OF MODELING APPROACHES

Almost all publications for review and analysis on the similar topic [1-19] offer a classification of the different approaches that have been developed to analyze and predict electricity prices, although they all have many things in common. Without rejecting the adoption of a generally accepted principle, in this article as a starting point, we will propose the segmented model classification differentiated in six groups of models. Additionally, it should be taken into consideration that an important direction of electricity price models originates from fundamental or structural electricity price models. For these models the electricity price is considered as an equilibrium of supply and demand (see e.g. [27-28]). The major price drivers are the fundamental inputs like the load and the merit order curve and especially the marginal cost of the available power plant portfolio. These fundamental models are popular for long term forecasting of electricity prices as impacts of regulative changes, for example the closing of a certain power plant or a newly installed wind farm, can be easily drawn.

These model types often lack to use realistic time series of prices and related data and therefore cannot provide a realistic hourly resolution of price predictions, which is typically the case in day-ahead markets. Nevertheless, there are some models which are able to capture the hourly behavior of electricity price and provide forecasts for time horizon of one month to one year as mid-term and to the time horizon of more than one year as long-term. The model of [29] for instance consists of a hybrid approach using fundamental and econometric, e.g. autoregressive, modeling techniques. They are able to utilize the hourly day-ahead electricity price series to forecast the whole year. It has been proved that Yan and Chowdhury were able to use data mining techniques, e.g. support vector machines (SVM), to study the PJM market in 2013 and 2015. In 2013 they showed by a forecasting study that combining a least squares support vector machine with an autoregressive-moving average with exogenous terms (ARMAX) model gives promising results for the consideration of one month hourly forecast.

However, from a practical standpoint it could be argued that the way how electricity prices are modeled is not as much important as the goal the modeling strategy actually pursues. For instance, if an electricity company is interested in building a new power plant, they will mainly be interested in long term electricity price forecasts over the whole lifetime of the plant, rather than to focus on a short-term horizon. Comparing different model strategies however may prove not to be too useful in this situation as it requires deep knowledge about properties and limitations of these models. Therefore, models can be differentiated according to their actual purpose of modeling rather than the model itself. Considering this and due to the declining popularity of the models based on the cost of electricity generation and the increasing use of simulation models, the first two groups of the presented models in this article can be combined into a larger class:

2.1. Multi-agent models (equilibrium supply function, Nash-Cournot, production-cost models, agent-based simulation, multi-agent simulation, general equilibrium, game theory) that simulate the functioning of a system of heterogeneous agents interacting with each other while building the pricing process considering the demand and supply on the market.

In the past, the forecasting process of wholesale electricity prices was labor-intensive task. The same, basically referred to the medium and long-term time horizons and included a comparison of the estimates of consumption and production obtained by arranging the existing and planned production units according to their operating costs. These Cost Based Models (Production Costs Models, PCM) have been able to predict prices per hour [25]. However, they ignore strategic bidding practices, including the use of market power. They are suitable for regulated markets with low price uncertainty, stable structure and no expected price volatility, but are not suited to competitive electricity markets. Theoretical approaches can be seen as summaries of cost models modified for strategic bidding considerations.

These models are useful in forecasting expected price levels in markets without price history, but with known cost of supply and market concentration. On the other hand, increasingly popular simulation techniques based on adaptive agents can respond to the characteristics of electricity markets that ignore static equilibrium patterns. Three main approaches for modeling of the electricity market trend are identified: optimization, equilibrium and simulation models. In their classification, optimization models focus on the problem of profits maximization for one of the market-competing firms. As such, they are not useful in the context of the EPF. The equilibrium models discussed below (Nash-Cournot, supply function equilibrium) represent the overall market behavior, taking into account the competition between all participants. Finally, simulation models are an alternative to equilibrium models when the problem under consideration is too complicated to address within a formal equilibrium framework. Since equilibrium and simulation models share many common features, they are considered together in a broad range of multi-agent models.

2.2. Fundamental structural methods that describe the dynamics of prices by modeling the impact of important physical and economic factors on the cost of electricity.

This class of models are usually known as fundamental or structural models and they are trying to capture the basic physical and economic ties that exist in the production and trading of electricity. Functional associations between fundamental management indicators (consumption, weather conditions, system parameters, etc.) are postulated by inputs that are modeled and predicted independently, often through statistical, reduced or computational AI techniques. In addition, many of the EPF approaches considered in the literature are hybrid solutions with time series, regression and neural network models as fundamental factors such as consumption, fuel prices and production resources, production technology

costs, wind power, or temperatures and weather forecasts - as input variables. In general, two subclasses of fundamental models can be identified: models rich in parameters and economical models based on supply and demand. [4]

2.3. Quantitative, stochastic forms of models that are characterized with the application of statistical data of electricity prices over time, with risk management purposes.

A common feature of this form of price-driven dynamics models is inspired by finance as their primary intention is not to provide accurate hourly price forecasts but rather to reproduce the main features of daily electricity prices as marginalized distributions at future time points, price dynamics and price correlation between raw materials. These models form the basis of derivatives pricing systems and risk management systems. If the chosen pricing process is not suited to capturing the main properties of electricity prices, the model results are likely to be unreliable. At the same time, if the model is too complex, the computational burden will prevent its online usage in the trade departments for practical purposes. On the one hand, the applied tools have their roots in methods that are designed to model other energy commodities or interest rates. On the other hand, they integrate processes to attract and improve decision-making by developing models to evaluate actions that will lead to unknown or econometric abrupt shifts in prices.

At this point, jump diffusion models and Markov's switching patterns offer the most optimal solution: they are compromise solution between the model of parsimony and adequacy to capture the unique characteristics of electricity prices. Depending on the type of the concerned market, stochastic techniques can be divided into two main classes: Spot and Forward models. The former represent the correct representation of spot price dynamics, which, following the deregulation of electricity markets is becoming a necessary tool for commercial purposes. Their main disadvantage is the problem of pricing of derivatives, that is, the identification of the risk premium linking spot and forward prices [14].

On the other hand, forward rate models allow pricing of derivatives in a simple way. However, they also have their limitations, among which the most important is the lack of data that can be used for calibration and also the inability to retrieve spot price properties from the analysis of forward supply and demand curves. Forward price models are derived and fall into mathematical finance, and therefore they will be subject of extensive discussions. Building smooth price curves on electricity markets can be challenging; but the benefits of this are easily accessible medium-term price forecasts for multiple horizons. However, these estimates may also include the risk premium.

2.4. Statistical approaches as econometric and technical analyses that are either direct application of statistical techniques to predict consumption or applications of econometric models in the electricity market.

Reduced-form models outweigh derivative valuation and risk analysis. However, when forecasting day-ahead electricity prices, simplicity and analytical flexibility of models is no longer an advantage. In fact, the simplicity of the model can be a serious constraint. Historically, the first input of statistical techniques of the EPF consisted mainly of statistical methods for predicting consumption. By simply replacing of prices for loads (and possibly loads for temperatures), the researchers were able to obtain EPF models.

Over time, more and more modern statistical econometric or signal processing techniques were introduced in this field. Methods for statistical (econometric, technical analysis) predict the current price using a mathematical combination of the previous prices and / or previous or current values of exogenous factors usually consumption and production data or weather variables. The two most important categories are additive and multiplicative models. They differ in whether the predicted price is the sum (additive) of a number of components or product (multiplied) by a number of factors. The first ones are much more popular. It should be noted that the two are closely related: a multiplicative model for pricing could become in a model of addition for logarithmic pricing. Statistical models can give some physical interpretation of their components, which allows engineers and system operators to understand their behavior. They are often criticized for their limited ability to model non-linear behavior of electricity prices and related basic fundamental variables; however, in practical applications their results are comparable to those of their non-linear alternatives.

2.5. Techniques based on computational intelligence as artificial intelligence, non-parametric, non-linear statistical techniques that combine elements of self-learning algorithms, evolution and fuzziness to create approaches that are adaptable to complex dynamic systems and can be seen as "intelligent".

Computational Intelligence (CI) is difficult to define because it means different things to different people. However, for CI may be considered that is a very diverse group of nature-inspired computational technique that have been developed to solve problems which traditional methods (e.g., statistical) cannot handle effectively. CI combines elements of self-learning, evolution, and blurring to create approaches capable of adapting to sophisticated dynamic systems, and in this sense can be seen as "intelligent".

Some authors use the term computer intelligence as synonym for artificial intelligence (AI), [26] as it almost everywhere in the EPF literature used. It should be noted that other names for AI techniques, such as non-parametric or non-linear statistics, may be found in the literature. However, these terms are too narrow or conflict with other classes of methods. For example, there are both non-parametric (e.g., kernel density estimator) and non-linear (threshold AR) techniques, which are usually classified as belonging to the group of statistical methods. Artificial neural networks, fuzzy systems, SVM and evolutionary computations (genetic algorithms, evolutionary programming and swarm intelligence) are

undoubtedly the main classes of AI techniques. Some authors also include probability reasoning and belief networks (at the intersection of traditional AI), artificial life techniques (biochemical intersection) and wavelets (at the intersection with digital wavelet processing). AI can also be linked to soft calculations, machine learning, data mining and cybernetics. AI models are flexible and can handle complexity and non-linearity.

This makes them promising for short-term forecasts and a number of authors report their excellent performance in the EPF. As with the prediction of consumption, artificial neural networks may have received the most attention. Other non-parametric techniques, such as fuzzy logic, genetic algorithms, evolutionary programming and swarm intelligence, but usually in hybrid constructs, are also used.

Also, it should be mentioned that many of the approaches to price modeling and forecasting that are considered in the literature are hybrid solutions combining techniques from two or more of the above-mentioned groups. Their classification is not trivial if it is even possible. The proposed structure is illustrated in Figure 1 with the main types of models discussed below.

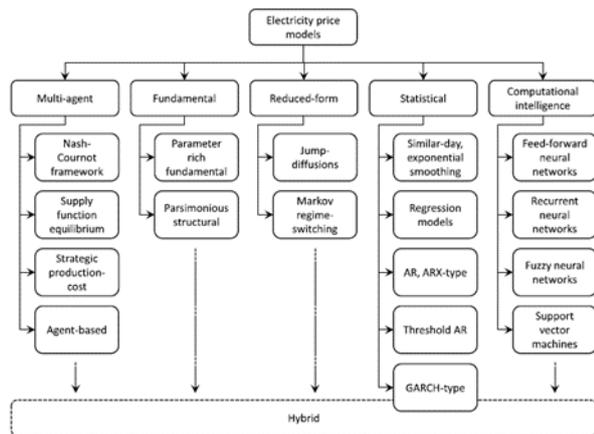


Figure 1: proposed structure of main types of EPF

3. CONCLUSIONS

The subject discussed in this paper was to present the different hybrid solutions, modeling and price forecasting approaches and combining techniques from more than one of the above-mentioned groups.

In the restructured electricity markets the application of electricity price forecasting techniques helps market participants and energy companies in their decision-making processes. Due to the nonlinear behavior of price dynamics and its dependence on many parameters, forecasting always faces unavoidable problems, mistakes and errors. In order to improve the forecasting of electricity prices in future more efficient instruments and their combination should be used.

The purpose of this paper was to present a short review the EPF methods and depending on the needs to provide the most appropriate method that should deliver the best results by combining several hybrid techniques. In order to provide satisfactory results and provide the best practice, the proposed methodologies should be tested with using the real data from the Bulgarian electricity market. It can be considered that due to high performance

and efficiency of the applied method in uncertainty modeling and high training accuracy, in the further researches, the clonal selection algorithm, extreme learning machine for neural networks training process and wavelet preprocess would be a good choice for short term probabilistic forecasting. In addition, for the best forecasting tool setup it should be taken into consideration the use of one year trading data to forecast the next year prices applying a two-stage SVM as an extension to the proposed modelling approach in order to be able to capture severe price peaks, which are described as extremely difficult to model in a midterm forecasting setting [30]. We hope that this report will give stimulation to those working in other areas of forecasting in order to move to the exciting, unique and largely unexplored field of the wholesale electricity market in Bulgaria.

REFERENCES

- [1]. Aggarwal, S. K., Saini, L. M., & Kumar, A. (2008). Electricity price forecasting in Ontario electricity market using wavelet transform in artificial neural network based model. *International Journal of Control, Automation and Systems*, 6(5), 639–650.
- [2]. Amjady N. Day-ahead price forecasting of electricity markets by a new fuzzy neural network. *IEEE Trans Power Syst* 2006;21(2):887–96.
- [3]. Benth, F. E., Benth, J. S., & Koekebakker, S. (2008). *Stochastic modeling of electricity and related markets*. Singapore: World Scientific.
- [4]. Burger, M., Graeber, B., & Schindlmayr, G. (2007). *Managing energy risk: an integrated view on power and other energy markets*. Wiley
- [5]. Catalão J, Mariano S, Mendes V, Ferreira L. Short-term electricity prices forecasting in a competitive market: a neural network approach. *Electr Power Syst Res* 2007;77(10):1297–304.
- [6]. Eydeland, A., & Wolyniec, K. (2003). *Energy and power risk management*. Hoboken, NJ: Wiley.
- [7]. Nowotarski J, Weron R. Computing electricity spot price prediction intervals using quantile regression and forecast averaging. *Comput Stat* 2015;30(3):791–803.
- [8]. Shahidehpour, M., Yamin, H., & Li, Z. (2002). *Market operations in electric power systems: forecasting, scheduling, and risk management*. Wiley. (2002, Chapter 3, pp. 57–113);
- [9]. Trück, S., Weron, R., & Wolff, R. (2007). Outlier treatment and robust approaches for modeling electricity spot prices. *Proceedings of the 56th Session of the ISI*. Available at MPRA: <http://mpra.ub.unimuenchen.de/4711/>.
- [10]. Wang, P., Zareipour, H., & Rosehart, W. D. (2014). Descriptive models for reserve and regulation prices in competitive electricity markets. *IEEE Transactions on Smart Grid*, 5(1), 471–479.
- [11]. Weron, R. (2006). *Modeling and forecasting electricity loads and prices: a statistical approach*. Chichester: Wiley.
- [12]. Weron, R., & Misiorek, A. (2005). Forecasting spot electricity prices with time series models. *IEEE Conference Proceedings – EEM05* (pp. 133–141).
- [13]. Weron, R., & Misiorek, A. (2006). Short-term electricity price forecasting with time series models: A review and evaluation. In W. Mielczarski (Ed.), *Complex electricity markets* (pp. 231–254). Łódź: IEPL& SEP.
- [14]. Weron, R., & Zator, M. (2014a). Revisiting the relationship between spot and futures prices in the

- Nord Pool electricity market. *Energy Economics*, 44, 178–190.
- [15]. Weron, R., Simonsen, I., & Wilman, P. (2004). Modeling highly volatile and seasonal markets: evidence from the Nord Pool electricity market.
- [16]. Yan, X., & Chowdhury, N. A. (2010a). Electricity market clearing price forecasting in a deregulated market: a neural network approach. VDM Verlag Dr. Müller.
- [17]. Yao, S. J., Song, Y. H., Zhang, L. Z., & Cheng, X. Y. (2000). Prediction of system marginal price by wavelet transform and neural network. *Electric Machines and Power Systems*, 28(10), 983–993.
- [18]. Zareipour, H. (2008). Price-based energy management in competitive electricity markets. VDM Verlag Dr. Müller.
- [19]. Zareipour, H., Janjani, A., Leung, H., Motamedi, A., & Schellenberg, A. (2011). Classification of future electricity market prices. *IEEE Transactions on Power Systems*, 26(1), 165–173.
- [20]. Rafał Weron. Electricity price forecasting: a review of the state-of-the-art with a look into the future. *Int J Forecast* 2014;30(4):1030–81.
- [21]. Aggarwal Sanjeev Kumar, Saini Lalit Mohan, Kumar Ashwani. Electricity price forecasting in deregulated markets: a review and evaluation. *Int J Electr Power Energy Syst* 2009;31
- [22]. Yan Xing, Chowdhury Nurul A. Mid-term electricity market clearing price forecasting: a hybrid LS SVM and ARMAX approach. *Int J Electr Power Energy Syst*
- [23]. Bello Antonio, Bunn Derek, Reneses Javier, Muñoz Antonio. Parametric density recalibration of a fundamental market model to forecast electricity prices. *Energies* 2016;9(11):959.
- [24]. Bello Antonio, Reneses Javier, Muñoz Antonio, Delgadillo Andrés. Probabilistic forecasting of hourly electricity prices in the medium-term using spatial interpolation techniques. *Int J Forecast* 2016;32(3):966–80.
- [25]. Wood, A.J. and Wollenberg, B.F. (1996) *Power Generation, Operation and Control*. John Wiley, New York.
- [26]. Poole, D., Mackworth, A., & Goebel, R. (1998). *Computational intelligence: a logical approach*. Oxford University Press.
- [27]. Lion Hirth. The market value of variable renewables: the effect of solar wind power variability on their relative price. *Energy Econ* 2013;38:218–36.
- [28]. Dillig Marius, Jung Manuel, Karl Jürgen. The impact of renewables on electricity prices in Germany-an estimation based on historic spot prices in the years 2011–2013. *Renew Sustain Energy Rev* 2016;57:7–15.
- [29]. Barquín Javier, Julián García-González, Dueñas Pablo. A hybrid approach fo modeling electricity price series in the medium term. In: *Proceedings of the power systems computation conference; 2008*.
- [30]. Yan Xing, Chowdhury Nurul A. Midterm electricity market clearing price forecasting using two-stage multiple support vector machine. *J Energy* 2015.