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Statistical Methods for Assessment of Latent-Structural Models in Robotics

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***Abstract*— The study presents a methodological approach for latent structural analysis (LSA) in robotics. LSA uses mathematical models to represent the relationship between latent variables and their indicators. The main models include a Y-measurement model for exogenous variables and an X-measurement model for endogenous variables. Various measures such as RMSEA, SRMR, CFI and TLI are used to check the adequacy of the models. RMSEA and SRMR assess model fit, with values below 0.08 considered good. The CFI and TLI also ranged between 0 and 1, with values above 0.90 indicating a good fit.**

***Keywords*— latent structural analysis (LSA), robotics, statistical methods, RMSEA, SRMR, CFI, TLI**

I. Introduction

Today's companies in the robotics sector are characterised by a highly dynamic structure, operating in unpredictable changes. They must make precise and clear decisions to develop a model to study their agility. Therefore, the present study aims to support managers in decision-making by applying a latent-structural approach to studying their organisational agility. The concept of agility has been widely debated in terms of modern project management concepts or the flexibility of manufacturing plants or systems [8], but we are not aware of it being used to study modern organisational types and in particular in the robotic sector. We must make a profound and large-scale study about the overall activity of these companies because, so far, they have mainly been used when considering agile projects of specific activities. Path, factor and latent structural analysis were applied in the study. Path analysis assesses the degree of equality and fit of a data set that is needed to adjust a theoretical model, such as a causal diagram [5]. In [10] a structural approach for organizational agility path analysis is proposed. A successful approach begins by surveying and identifying factor dependencies from the literature relevant to path analysis.

Therefore, as an object of research, methods for evaluating models in latent-structural analysis were chosen. In connection with their clarification, basic notations used in the LSA methodology are attached. The subject of research is how models will be evaluated in LSA.

Therefore, the present study aims to present statistical approaches for estimating models in LSA. Mathematical formulas for each of the models are provided, as well as the matrix form of the measurement models. Measures are also given to check the adequacy of the models, which shows how well they correspond to reality. To achieve the goal, it is necessary to solve the following tasks:

1) To be considered various ways of application of latent structural analysis in robotics;

2) To present the mathematical formulas for latent-structural analysis;

3) To propose metrics for checking the adequacy of the model.

II. Application of Latent Structural Analysis in Robotics

Latent Structural Analysis (LSA) can be applied in various ways within the field of robotics. Some key applications are:

1) *Structural Optimization:* LSA helps in optimizing the design of robotic structures by analyzing stress distribution and identifying weak points. This ensures that robots are both lightweight and durable, improving their performance and longevity [14].

2) *Dynamic Analysis:* By using LSA, engineers can perform dynamic analysis of robotic components, such as gears and joints, to predict their behavior under different operating conditions. This is crucial for ensuring the reliability and efficiency of robots in various tasks [1].

3) *Path Planning and Control:* LSA can be integrated with machine learning techniques, such as variational autoencoders (VAEs), to enhance path planning and control in robots. This allows robots to navigate complex environments more effectively [11].

4) *Route optimization:* In robotics, especially in autonomous vehicles, LSAs can help optimize routes by solving linear optimization problems. Reference [8] offers FMS parts flow path analysis.

5) *Rehabilitation Robotics:* In the design of exoskeletons and other assistive devices, LSA is used to ensure that these devices can support human movement accurately and safely. This is particularly important for rehabilitation robots that assist individuals with mobility impairments [14].

These applications demonstrate how LSA contributes to the advancement of robotics by improving design, functionality, and adaptability.

Structural optimization using Latent Structural Analysis (LSA) involves several key steps and benefits:

1) *Stress and Strain Analysis:* LSA helps in identifying areas within a robotic structure that experience high

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stress and strain. By analyzing these areas, engineers can redesign components to distribute loads more evenly, reducing the risk of failure.

2) *Material Efficiency:* By understanding the structural demands, LSA allows for the use of materials more efficiently. This means that robots can be made lighter without compromising their strength, which is crucial for improving energy efficiency and performance.

3) *Topology Optimization:* LSA can be used to optimize the topology of robotic components. This involves creating structures that are not only strong but also use the least amount of material possible. This is particularly useful in additive manufacturing (3D printing), where material savings can lead to significant cost reductions.

4) *Fatigue Analysis:* LSA helps in predicting the lifespan of robotic components by analyzing how they will behave under repeated loading and unloading cycles. This is essential for ensuring that robots can operate reliably over long periods.

5) *Dynamic Performance*: By optimizing the structural design, LSA can improve the dynamic performance of robots. This includes better handling of vibrations and impacts, which is important for robots operating in dynamic environments.

6) *Customization for Specific Tasks:* LSA allows for the customization of robotic structures for specific tasks. For example, a robot designed for heavy lifting can be optimized differently than one designed for precision tasks.

These optimizations lead to more robust, efficient, and cost-effective robotic systems.

III. Mathematical approaches for latent structural analysis

Before The mathematical apparatus for LCA is a tool used to represent the relationship between latent variables and indicators [1]. This can be expressed as follows:

$$y_i = \Lambda_x \eta_i + e_i \tag{1}$$

$$x_i = \Lambda_y \xi_i + \delta_i \tag{2}$$

Model (1) is a Y-dimensional model in which the corresponding indicator variables describe the exogenous variables. It can be represented in matrix form as follows:

$$\begin{pmatrix} \boldsymbol{y_{i1}} \\ \vdots \\ \boldsymbol{y_{iq}} \end{pmatrix}_{\boldsymbol{q\times 1}} = \begin{pmatrix} \boldsymbol{\lambda_{11}} & \cdots & \boldsymbol{\lambda_{1n}} \\ \vdots & \vdots & \vdots \\ \boldsymbol{\lambda_{q1}} & \cdots & \boldsymbol{\lambda_{qn}} \end{pmatrix}_{\boldsymbol{q\times n}} \begin{pmatrix} \eta_{i1} \\ \vdots \\ \eta_{in} \end{pmatrix}_{\boldsymbol{n\times 1}} + \begin{pmatrix} \epsilon_{i1} \\ \vdots \\ \epsilon_{iq} \end{pmatrix}_{\boldsymbol{q\times 1}}, \tag{3}$$

Model (2) is an X-dimensional model in which the corresponding indicator variables describe the endogenous variables. It can be represented in matrix form as follows:

$$\begin{pmatrix} x_{i1} \\ \vdots \\ x_{ip} \end{pmatrix}_{p\times 1} = \begin{pmatrix} \lambda_{11} & \cdots & \lambda_{1m} \\ \vdots & \ddots & \vdots \\ \lambda_{p1} & \cdots & \lambda_{pm} \end{pmatrix}_{p\times m} \begin{pmatrix} \boldsymbol{\xi}_{i1} \\ \vdots \\ \boldsymbol{\xi}_{im} \end{pmatrix}_{m\times 1} + \begin{pmatrix} \boldsymbol{\delta}_{i1} \\ \vdots \\ \boldsymbol{\delta}_{ip} \end{pmatrix}_{p\times 1}, \tag{4}$$

where δ_i and ϵ_i are typically distributed independent variables.

IV. Metrics for checking the adequacy of the model

A. *Root Mean Squared Error of Approximation/ Root Mean Squared Error of Approximation (RMSEA)*

The results of this study suggest that model fit studies, in the presence of large sample sizes, can be supplemented by applying the RMSEA statistic. RMSEA values less than <0.02 with sample sizes of 500+ and even at 1000+ can undoubtedly indicate that the data do not fit the model and that the X-square is inflated with sample size.

RMSEA is a measure of empirical/absolute goodness of fit/correctness used [4] and its value ranges from 0 to 1. Reference [7] suggests an acceptable value of RMSEA to be between 0.05 and .08 to have a reasonable well-fitting model. Statistically, this indicator can be expressed as follows:

$$\text{RMSEA} = \left[\frac{(x^2)-df}{df(n-1)}\right]^{0.5} : \text{with} x^2 = (n-1)Fmin, \tag{5}$$

Where *df* is the model degree of freedom and *Fmin* is the minimum value of the fitness function of the estimation method used. Despite the popularity of this fit index in LSA studies, simulation studies in the literature have concluded that RMSEA does not perform well because it too often rejects the actual model at small sample sizes (n<250), and its value can get worse as the number of variables increases in the model. According to the statement [11], the following measure, SRMR, is recommended over RMSEA.

B. *Standardized Root Mean Square Residual/ Standardized Root Mean Square Residual (SRMR).*

SRMR has similar properties to RMSEA indices but is calculated differently and shows a poor model fit with higher values, while a good model fit would be an SRMR value close to zero. However, [9] suggests an SRMR value of less than 0.08, indicating a good model fit. Statistically, this indicator can be expressed as follows:

$$\text{SRMR} = \left[\frac{\sum_{i=1}^{p}\sum_{j=1}^{i}\left[(s_{ij}-\hat{\sigma}_{ij})/(s_{ii}s_{jj})\right]^2}{k(k+1)/2}\right]^{0.5}, \tag{6}$$

Where $k = p + q$, s_{ij}, and $\hat{\sigma}_{ij}$ are the sample covariance between the observed variables, and are the estimated components of the variance-covariance matrix of the model error vector.

C. *Comparative Fit Index (CFI)*

The CFI value ranges between 0 and 1, with a value closer to 1 indicating a better fit. [8] recent studies suggest that a CFI value above 0.95 is considered an indicator of good model fit or at least 0.90 or higher to ensure that the model is accurately represented. The formula used to calculate the CFI is expressed as follows:

$$\mathrm{CFI} = 1 - \frac{Max\left((x^2_{model} - df_{model}),0\right)}{Max(x^2_{null} - df_{null}),0)}, \tag{7}$$

Here, Max indicates the maximum value of the expressions given in parentheses. The comparison between the model's x^2 and its degrees of freedom is considered the bias correction of the model.

D. *The Tucker-Lewis Index (TLI)*

The TLI index was introduced by [3]. It is also known as the non-normed Fit Index (NNFI). Its value also varies between 0 and 1, with a value closer to 1 indicating a better fit for the model. [9], Suggest a value of 0.95 or higher as an indicator of a well-structured model. The formula by which this index is calculated can be expressed as follows:

$$\mathrm{TLI} = \frac{(x^2_{null}/df_{null}) - (x^2_{model}/df_{model})}{(x^2_{null}/df_{null}) - 1}, \tag{8}$$

Where x^2_{null}/df_{null} is the ratio of the x^2 to the degree of freedom *df.*

TABLE 1. CRITERIA FOR ADEQUACY OF THE MODEL

Criterion	**Value**
Incremental Fit Index (IFI)	0.820
Relative Fit Index (RFI)	0,700
Comparative Fit Index (CFI)	0.816
Normed Fit Index (NFI)	0.776
Non-Normed Fit Index (NNFI)	0.753
Parsimony Normed Fit Index (PNFI)	0,579
RMSEA	0,153
Root Mean Square Residual (RMSR)	0,219
Adjusted Goodness of Fit Index (AGFI)	0.697
Parsimony Goodnes of Fit Index (PGFI)	0,504
Standardized RMR	0,0775
GFI	0.812

According to [2], variables whose coefficients of direct influence are less than ± 0.1 are excluded from the model. Using the results of the path analysis, the relationship of the model indicators can be defined directly and indirectly, and the relative importance of the relationships of the direct and indirect variables can be assessed using a software package such as LISREL, for example [5]. In [12], path analysis for customer experience evaluation of a virtual gaming platform is conducted.

The study presents several methods and approaches for latent structural analysis. LSA's mathematical apparatus is a powerful tool for representing the relationship between latent variables and their indicators. Measurement models for exogenous and endogenous variables can be represented in matrix form, with customarily distributed independent errors playing a pivotal role.

Numerous measures were considered to check the adequacy of the model. The root mean square standard error of approximation (RMSEA) is an essential measure of empirical fit, with values below 0.05 to 0.08 considered acceptable. However, with small sample sizes, RMSEA may not perform well. The standardised root mean square residual (SRMR) shows good model fit at values close to zero. Values below 0.08 are considered good. The comparative fit index (CFI) ranges between 0 and 1, with values above 0.90 indicating good model fit. The Tucker-Lewis Index (TLI) also varies between 0 and 1, with values above 0.95 indicative of a well-structured model. These metrics provide different perspectives for evaluating model adequacy and can be used more comprehensively in evaluating LSA models.

V. CONCLUSION

This study presents numerous methods and approaches to latent structural analysis (LSA). LSA's mathematical apparatus is a powerful tool for representing the relationship between latent variables and their indicators. Measurement models for exogenous and endogenous variables can be represented in matrix form, with customarily distributed independent errors playing a pivotal role.

Several measures were considered to check the adequacy of the model. The root mean square standard error of approximation (RMSEA) is an essential measure of empirical fit, with values below 0.05 to 0.08 considered acceptable. However, with small sample sizes, RMSEA may not perform well. The standardised root mean square residual (SRMR) shows good model fit at values close to zero. Values below 0.08 are considered good. The comparative fit index (CFI) ranges between 0 and 1, with values above 0.90 indicating good model fit. The Tucker-Lewis Index (TLI) also varies between 0 and 1, with values above 0.95 indicating a well-structured model. These metrics provide different perspectives for evaluating model adequacy and can be used together to evaluate LSA models comprehensively.

In conclusion, structural optimization using Latent Structural Analysis (LSA) significantly enhances the design and functionality of robotic systems. By meticulously analyzing stress, strain, and material efficiency, LSA enables the creation of lightweight yet robust structures. This optimization not only improves the dynamic performance and longevity of robots but also ensures cost-effective and sustainable manufacturing processes. Ultimately, LSA empowers engineers to design highly specialized and efficient robots tailored to specific tasks, paving the way for advanced and reliable robotic solutions in various industries.

As a result of this study, it is concluded that the study of LSA is crucial for the adequate functioning of any modern robotic enterprise. These methods can be used in robotics, by implementing them in intelligent systems, and thus help improve the agility of robotic manufacturing.

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