Longitudinal Stability of Wheeled Mobile Robots with Movable Center of Mass

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***Abstract*— This paper is a study of the longitudinal stability of a wheeled mobile robot with a variable center of mass position depending on the longitudinal position of the static component of the center of mass and the range of longitudinal movement of its movable component. The calculations are based on D'Alembert's principle. The results represent values of the permissible driving/braking force and permissible acceleration/deceleration for a given position of the center of mass at which the robot keeps stability.**

Keywords—wheeled mobile robots, longitudinal stability, permissible forces and accelerations, movable center of mass

I. INTRODUCTION

The problems addressed in the paper are in the field of mobile robotics. They are about achieving a reasonable compromise between stability and controllability when movement.

By increasing the wheelbase of a wheeled robot, greater stability is achieved in the longitudinal direction, but this also leads to an increase in the required turning time. The question here is whether a variable center of mass position can be used to increase the longitudinal stability of the robot instead of achieving it by increasing the wheelbase

Usually, in similar studies, differential equations are used [1], [2], [4], [5] to represent the mathematical model. Instead, the principle of kinetostatics is used here, which considers an equilibrium system of forces, including inertial forces [6]; the accuracy of the calculations does not deteriorate.

The aim of the study is to determine the limit traction/braking forces and limit accelerations/decelerations at a variable position of the robot's center of mass, at a given mass and geometric proportions of the robot and to compare at what wheelbase length and fixed center of mass position, these limit forces and accelerations can be achieved. To achieve the goal, the following tasks are set:

- To draw a scheme of the experimental setup, which includes a structural scheme of a four-wheeled mobile robot and the forces acting on it during acceleration/deceleration.
- To determine a range in the longitudinal direction in which the position of the center of mass can move.
- To determine what percentage of the total mass of the robot is the movable mass, through which the longitudinal position of the center of mass can be influenced.
- Formulate equations to find the limit traction/braking force and limit acceleration/deceleration at which the robot begins to lose stability.

The hypothesis is that certain values of the limit longitudinal forces and accelerations can be achieved with a shorter base if the robot is constructed with a variable position of the center of mass in the longitudinal direction and this feature of the construction is used for balancing.

A criterion for loss of stability is the occurrence of a zero or negative value of the support reaction for any of the wheels of the robot.

II. METHODS AND MATERIALS

A. Brief theory

In Fig. 1 shows a longitudinal projection of the researched type of robot, which has rear driving wheels.

The total mass and position of the robot's center of mass is formed by the following components, which may be included in the structure and, possibly, the load:

- Body;
- Suspension;

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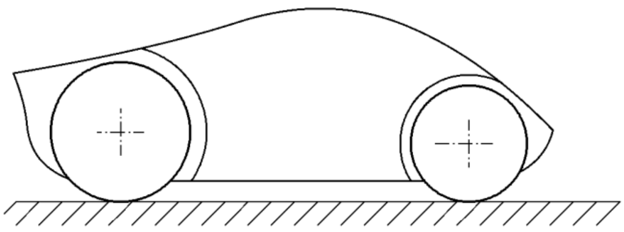

Fig. 1. Schematic of the robot in general view

- Wheels;
- Engine(s);
- Transmission;
- Equipment;
- Accumulator battery, which can be of considerable mass if the motors are electric;
- Payload;
- Fuel tank;
- Possibly ballast.

The mass/center of mass may possess a stationary, m_{cs} and movable component – m_{cm} (Fig. 2). For example, if ballast is included in the total mass, and it is mounted with the ability to move, it can be used to change the position of the center of mass, for the purpose of balancing during movement or positioning. Instead of ballast, however, if the construction allows it, one of its components can be used. In this way, the mass of the ballast will be subtracted from the total mass and the robot will be lighter. For example, in an electric drive, the battery has considerable mass, and this allows it to be used for balance, as long as a mechanism is provided to move it quickly.

According to [3], the mass of the battery of an electric car is 30 – 35% of its total mass. The ratio of the masses of the battery and the total mass of the robot for this study is assumed to be 0.33; that is, if the robot's battery is in use for a moving mass along its longitudinal axis, it $m_{cm} = 0.33m_c$.

On the other hand, the displacement of the battery for balancing purposes is limited by its adjacent robot components. For the considered structure, it is assumed that the displacement range of the moving component of the center of mass is $x_{m_{cm}} \epsilon [x_{mcs} - l_r/2; x_{mcs} + l_f/2]$, when $x_{m_{cs}} = const \equiv x_{m_{c_0}} \equiv x_{m_{cm_0}}$.

The notations used in the mathematical model and those of fig. 2, are as follows:

- Oxz – coordinate system related to the body of the robot;
- m_c – center of mass of the robot;
- m_{cs} – static component of the center of mass;
- m_{cm} – movable component of the center of mass;
- A_r – position of the support points of the rear wheels;
- A_f – position of the support points of the front wheels;
- $l = 0.5\ [m]$ – wheel base;
- l_r – distance along the x axis from the center of mass to the support points of the rear wheels;
- l_f – distance along the x axis from the center of mass to the support points of the front wheels;
- $h_{mc} = 0.1\ [m]$ – distance from the mass center to the road;
- $m = 3\ [kg]$ – mass of the robot;
- $g = 9.807\ [m/s^2]$ – average ground acceleration (in the general case, the gravitational acceleration is a parameter);
- a – acceleration of the robot;
- a_{tr} – positive acceleration of the robot;
- a_{br} – deceleration of the robot;
- a_p – limit acceleration of the robot;
- a_{trp} – limit positive acceleration of the robot;
- a_{brp} – limit deceleration of the robot;
- F_g – weight;
- F_{tr} – traction force;
- F_{br} – braking force;
- F_{trp} – limit traction force;
- F_{brp} – limit braking force;
- F_{in} – inertia force;
- F_{sr} – support reaction at the rear wheels;
- F_{sf} – support reaction at the front wheels;
- F_{rr}; F_{rf} – frictional forces during rolling, on rear and front wheels, respectively.

The mathematical model of the robot is built according to the principle of kinetostatics, for which in this case it is necessary to compile a system of two moments and one projection equation according to the setup in Fig. 2:

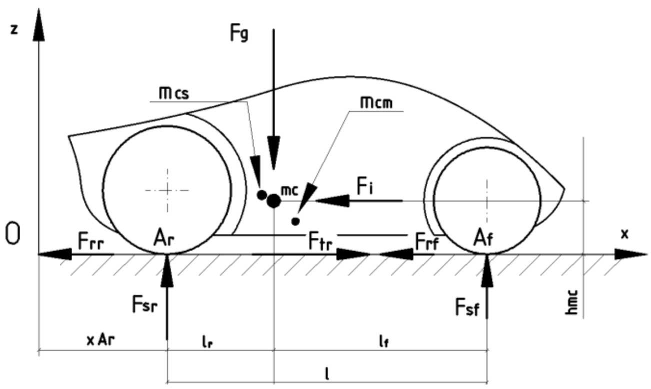

Fig. 2. Schematic of the robot with a coupled coordinate system and the acting forces (longitudinal projection)

$$\left| \begin{array}{l} \sum M_{A_{r_i}} = 0 \\ \sum M_{A_{f_i}} = 0 \\ \sum x_i = 0 \end{array} \right. \quad (1)$$

$$\left| \begin{array}{l} F_g\, l_r + F_i\, h_{m_c} + F_{s_f}\, l = 0 \\ F_g\, l_f + F_i\, h_{m_c} + F_{s_r}\, l = 0 \\ F_{tr} + F_{in} = 0 \end{array} \right. \quad (2)$$

If the robot is equipped with the necessary sensors for reading the traction force and an actuator for supplying the required traction force F_{tr}, i.e. if F_{tr} is a parameter, the support reactions of the wheels remain unknown in the system. The remaining terms in the equations are either constants or parameters:

$$F_g = m\, g \quad (3)$$

$$l = x_{A_f} - x_{A_r} \quad (4)$$

The following dependence determining the starting position of the center of mass is assumed:

$$l_r = \frac{1}{3} l;\ l_f = \frac{2}{3} l; (l = l_r + l_f) \quad (5)$$

For the support reactions, with positive acceleration, we get:

$$F_{s_f} = \frac{F_{tr}\, h_{m_c} - m\, g\, l_r}{l} \quad (6)$$

$$F_{s_r} = \frac{F_{tr}\, h_{m_c} + m\, g\, l_f}{l} \quad (7)$$

For the support reactions, at negative acceleration, we get:

$$F_{s_f} = \frac{F_{br}\, h_{m_c} + m\, g\, l_r}{l} \quad (8)$$

$$F_{s_r} = \frac{m\, g\, l_f - F_{br}\, h_{m_c}}{l} \quad (9)$$

According to the selected loss of stability criterion, in order to find the limit driving and braking forces, under acceleration/deceleration, we assume the support reactions to be zero in the following two equations:

$$F_{tr_p} = \frac{m\, g\, l_r - F_{s_f}\, l}{h_{m_c}} \quad (10)$$

$$F_{tr_p} = \frac{m\, g\, l_f - F_{s_r} l}{h_{m_c}} \quad (11)$$

Because $F_{trp} = m a_{trp}$; $F_{brp} = m a_{brp}$, then again for support reactions equal to zero:

$$a_{trp} = \frac{m\, g\, l_r - F_{s_f}\, l}{h_{m_c}\, m} = \frac{g\, l_r}{h_{m_c}} - \frac{F_{s_f}\, l}{h_{m_c}\, m} \quad (12)$$

$$a_{brp} = \frac{m\, g\, l_f - F_{s_r}\, l}{h_{m_c}\, m} = \frac{g\, l_f}{h_{m_c}} - \frac{F_{s_r}\, l}{h_{m_c}\, m} \quad (13)$$

B. *Implementation*

For this study, for design reasons, it is assumed that the moving component of the center of mass can move in the range $x_{m_{cm}} \epsilon [x_{mcs} - l_r/2; x_{mcs} + l_f/2]$. Given that $m_{cm} = 0.33 m_c$ and at the specified range for the moving component, the position of the center of mass along the axis x is:

$$x_{mc} = \frac{\sum m_i x_i}{\sum m_i} = \frac{x_{m_{cm}} + 2 x_{m_{cs}}}{3} \quad (14)$$

If in (14) the minimum and maximum values from the range of are entered $x_{m_{cm}}$, the movement of the general center of mass will be in the range:

$$x_{m_c} \epsilon [x_{mcs} - l_{r_0}/6; x_{mcs} + l_{f_0}/6], (x_{m_{cs}} = const \equiv x_{m_{c_0}}).$$

Experiments are conducted for acceleration and deceleration.

The steps in performing the calculations are as follows:

- Values for the constants are determined.
- The ground acceleration g, mass m, base, $l$and height of the center of mass h_{mc} remain the same for all calculations.
- The parameters are defined:
  - For acceleration calculations, l_r takes six values: 0.167; 0.183; 0.200; 0.217; 0.233; 0.250 [m].
  - For deceleration calculations, l_r takes the values: 0.167; 0.133; 0.100; 0.067; 0.033; 0.000 [m].
  - For acceleration calculations, l_f takes six values: 0.333; 0.317; 0.300; 0.283; 0.267; 0.250 [m].
  - For deceleration calculations, l_f take the values: 0.333; 0.367; 0.400; 0.433; 0.467; 0.500 [m].
  - The distance between the wheel axles and the center of mass in initial position ($x_{m_{cs}} \equiv x_{m_{cm}}$) are dependent: $l_r = \frac{1}{3} l;\ l_f = \frac{2}{3} l, (l = l_r + l_f)$.
- The force of gravity is determined according to the values of the earth's acceleration and mass: $F = mg$.
- For positive acceleration, the limit traction force is calculated by the equation: $F_{tr_p} = m g l_r / h_{mc}$.
- For deceleration, the limit braking force is calculated using the equation: $F_{br_p} = m g l_f / h_{mc}$.
- The limit positive acceleration is $a_{tr_p} = g l_r / h_{mc}$.
- The limit deceleration is $a_{br_p} = g l_f / h_{mc}$.

I. EXPERIMENTS AND RESULTS

A. *Subject of experiments*

The experiments are carried out on a design scheme of a four-wheel mobile robot with rear drive, with a variable position of the center of mass in the longitudinal direction.

B. Restrictions

The range of motion of the moving component of the center of mass is:

$$x_{m_{cm}} \epsilon [x_{mcs} - l_r/2; x_{mcs} + l_f/2].$$

The wheels contact the road at a point. Rolling friction forces are neglected. It is assumed that the motion occurs without slipping. It is also assumed that the robot body and its wheels are perfectly rigid, i.e. no deformations during movement.

C. Results

The results are presented in graphic form (Fig. 3, Fig. 4, Fig. 5, Fig. 6).

The first experiment is during acceleration, and the second – during braking. Driving wheels are the rear, and at limit traction force/limit acceleration, the support reactions at the front wheel axle tend to zero. During braking, the front wheel axle resistances increase and at limit braking force/limit deceleration, the rear wheel axle resistances tend to zero, i.e. in this case, the braking force acts exclusively through the front wheels.

The studied construction scheme has a wheel base $l = 0.5\ [m]$ and a movable center of mass with a range of motion $x_{m_c} \epsilon [x_{mcs} - l_{r_0}/6; x_{mcs} + l_{f_0}/6]$, $(x_{m_{cs}} = const \equiv x_{m_{c_0}})$.

II. Conclusion

The longitudinal stability of a four-wheeled mobile robot was studied depending on the longitudinal position of its movable center of mass. For certain positions of the center of mass, the limit force and the limit acceleration at which the robot loses longitudinal stability have been calculated.

Characteristic points represent the limit forces and limit accelerations when the moving component of the center of mass is in the extreme front and extreme rear positions.

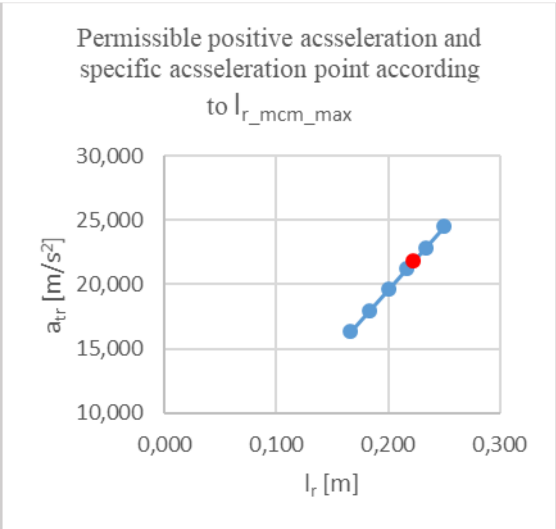

Fig. 3. The value of l_r at the specific acceleration point (red color) is 0.222 [m]

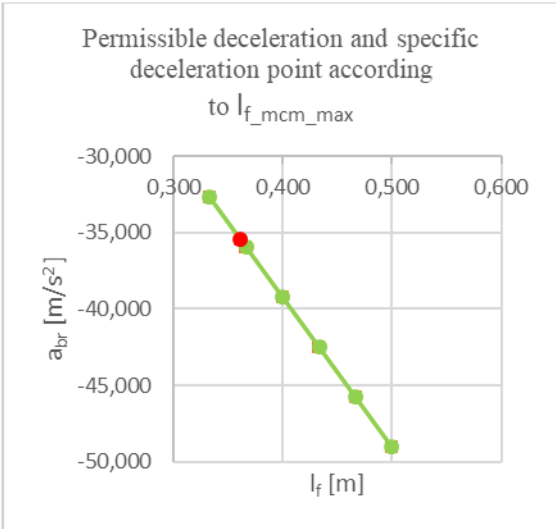

Fig. 5. The value of l_f at the specific deceleration point (red color) is 0.361 [m]

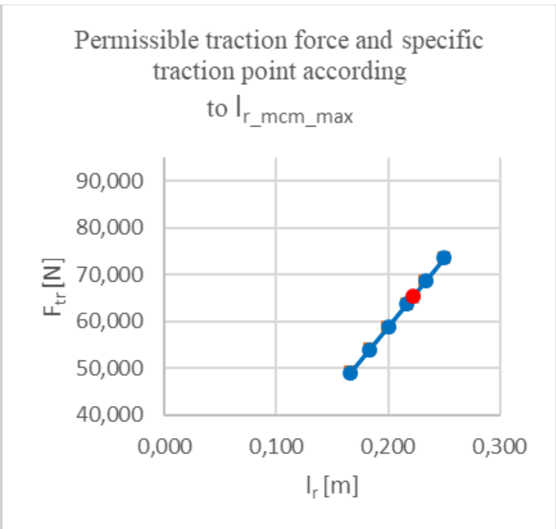

Fig. 4. The value of l_r at the specific traction point (red color) is 0.222 [m]

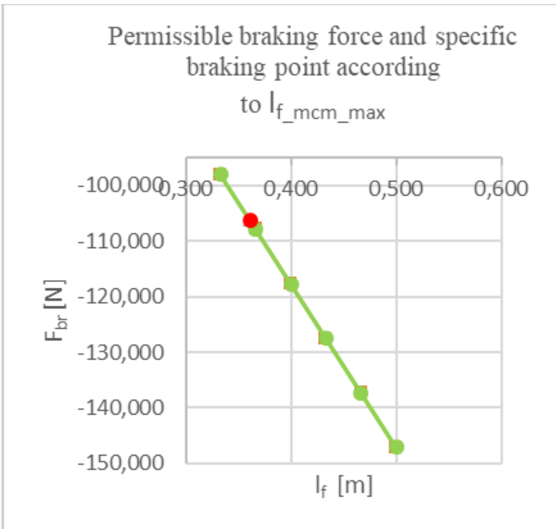

Fig. 6. The value of l_f at the specific braking point (red color) is 0.361 [m]

The studied construction scheme has a wheel base

$l = 0.5\ [m]$ and a movable center of mass with a range of motion
$x_{m_c} \epsilon [x_{mcs} - l_{r_0}/6; x_{mcs} + l_{f_0}/6]$; $(x_{m_{cs}} = const \equiv x_{m_{c_0}})$.

In order to compare this design scheme with a fixed center of mass scheme, in terms of limit longitudinal forces and accelerations, the wheelbase l_{fix} of the fixed center of mass scheme must be a sum of $l_{r\ max}$ and $l_{f\ max}$, established in the experimental part of the article, i.e. $l_{fix} = l_{r\ max} + l_{f\ max} = 0.222 + 0.361 = 0.583\ [m]$. Thus, in the specific case, the base l_{fix} is with 16.6% longer than l. Therefore, the use of a movable center of mass in the construction of wheeled mobile robots would be a suitable approach to achieve more compact dimensions, better maneuverability and possibly a lighter construction.

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