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Model Predictive Based Trajectory Tracking Control

Stoyan Lilov
Bulgarian Academy of Sciences
Institute of Roboticsl
Plovdiv, Bulgaria
0009-0002-5055-053
lsv@abv.bg

Vanya Markova
Bulgarian Academy of Sciences
Institute of Roboticsl
Plovdiv, Bulgaria
0000-0003-2648-5731
markovavanya@yahoo.comg

Ventseslav Shopov
lBulgarian Academy of Sciences
Institute of Roboticsl
Plovdiv, Bulgaria
0000-0001-8216-5914
vkshopov@yahoo.com

***Abstract*— The paper discusses the application of Model Predictive Control (MPC) to the task of tracking a predefined trajectory by an autonomous mobile robot with non-holonomic constraints. The mathematical model of a wheeled mobile robot is studied, an optimization problem is formulated to minimize the error in tracking the trajectory in the presence of speed and controllability constraints, as well as the influence of the prediction horizon on the accuracy and stability of the control. Simulation results are presented, which show that MPC offers high accuracy and good robustness under dynamic conditions, which makes it a suitable method for controlling autonomous systems in a real environment.**

Keywords: Model Predictive Control (MPC), trajectory tracking, autonomous robots, control systems.

I. INTRODUCTION

Autonomous mobile robots are a key element in modern intelligent systems used for transportation, logistics, mapping, inspection and cooperative tasks. A major problem in controlling such systems is tracking a given trajectory in the presence of constraints in the kinematics and dynamics, known as non-holonomic constraints. In this article, the focus is on wheeled mobile robots, but some of the calculations can also be used for other types, such as walking robots [7]. Non-holonomic systems, such as differentially driven robots (such as unicycle), cannot move laterally and their control requires consideration of nonlinear kinematic dependencies. Classical PID or linear controllers are often not sufficient to provide high accuracy and stability in real time, especially in the presence of obstacles, noise or dynamic disturbances [1].

Model Predictive Control (MPC) is a modern optimization approach that predicts the future dynamics of the system in real time, solves an optimization problem with constraints and selects the optimal control to minimize the error. The advantage of MPC is its flexibility – it can work with nonlinear models, physical constraints and different optimization criteria [2], [3] . Simulations are performed using the Rectangle method (RE) and Method of the surrounding pyramidal surface (PD) methods [9], [10]. The present work aims to present the application of MPC for trajectory tracking of an autonomous wheeled mobile robot with non-holonomic constraints, focusing on the mathematical formulation, the predictive control methodology and the simulation results.

II. MATERIALS AND METHODS

A. Kinematic model

The non-holonomic model of a mobile robot type unicycle is described by the equations:

$$\dot{x} = v\cos(\theta), \quad \dot{y} = v\sin(\theta), \quad \dot{\theta} = \omega \tag{1}$$

where x, y are coordinates, θ is the orientation, v is the linear velocity, and ω is the angular velocity. The non-holonomy constraint is:

$$\dot{y}\cos(\theta) - \dot{x}\sin(\theta) = 0 \tag{2}$$

1) Discretization

When discretizing with period T_s we get:

$$\begin{aligned} x_{k+1} &= x_k + T_s v_k \cos(\theta_k) \\ y_{k+1} &= y_k + T_s v_k \sin(\theta_k) \\ \theta_{k+1} &= \theta_k + T_s \omega_k \end{aligned} \tag{3}$$

2) Tracking error

For a given desired trajectory(x_d, y_d, θ_d):

$$e_x = x_d - x, \quad e_y = y_d - y, \quad e_\theta = \theta_d - \theta \tag{4}$$

The restrictions on control are:

$$v_{min} \le v_k \le v_{max}, \quad \omega_{min} \le \omega_k \le \omega_{max} \tag{5}$$

B. Bicycle model

When studying the kinematics of four-wheeled mobile robots with an Ackermann-type control system, the bicycle model Fig.1, Fig. 2 can be used to simplify calculations [8]. The bicycle model describes only the kinematics of the robot.

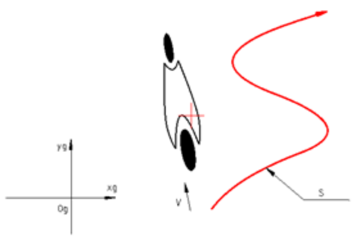

Fig. 1 Four-wheeled robot presented in a simplified bicycle scheme (top view)

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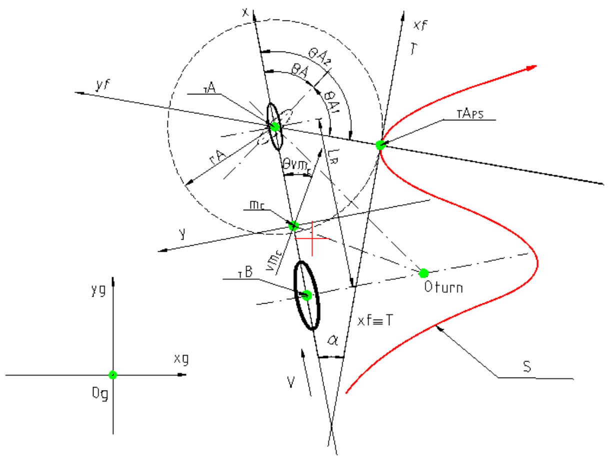

Fig. 2 A robot represented in a bicycle scheme approaching a given trajectory (top view)

C. *Model using D'Alembert's principle (RE model)*

In this model, the principle of D'Alembert is used for an equilibrium system of forces, including inertial ones Fig. 3. In the case of a wheeled mobile robot, a system of three equations is compiled, two moments and one projection, from which the values of the support reactions at the wheels are obtained. The criterion for loss of stability in this model is the appearance of a zero value for at least one of the support reactions [9].

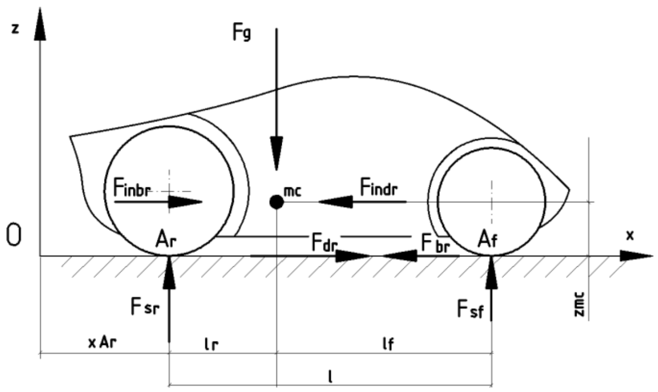

Fig. 3 Scheme of the forces acting on the robot, used in the RE method

D. *Pyramidal model (PD)*

In the pyramidal model Fig. 4, Fig. 5, Fig. 6, Fig. 7, the criterion for loss of stability during movement is reduced to the fact that a representative of the resultant vector of inertial accelerations, starting at the center of mass, ends up outside the pyramidal surface formed by the center of mass (as a peak) and the contact points of the wheels with the terrain of movement [10].

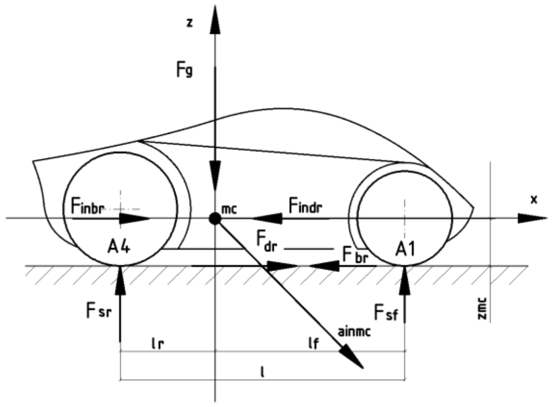

Fig. 4 Diagram of a robot and the resultant inertial vector acting on its center of mass (side view)

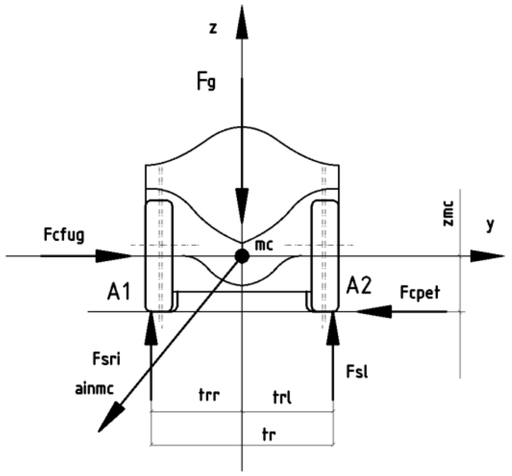

Fig. 5 Diagram of a robot and the resultant inertial vector acting on its center of mass (front view)

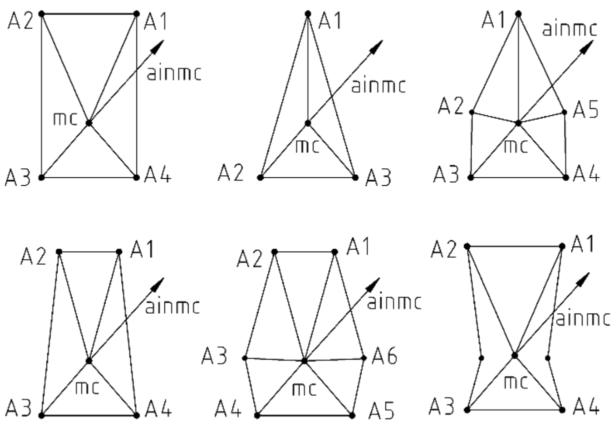

Fig. 6 Some possible configurations of the contact points of the robot's wheels with the terrain of movement (top view)

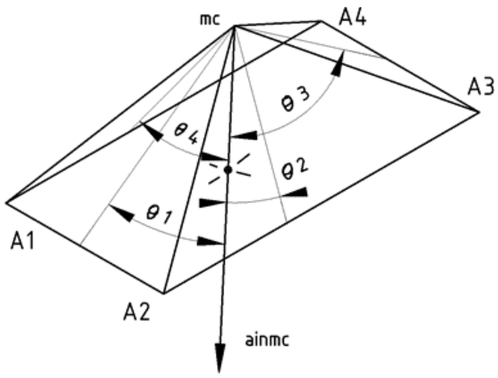

Fig. 7 Axonometric view depicting the spatial location of the robot's center of mass and the contact points of its wheels with the terrain of movement

E. *A brief review of model predictive control (MPC)*

Model Predictive Control (MPC) uses a dynamic model of the system to predict its future behavior within a prediction horizon of N_p steps ahead.

1) Optimization problem

At each iteration, an optimization problem is solved to minimize a functional of the form:

$$J = \sum_{i=1}^{N_p}\left[Q_e\left(e_{x,i}^2 + e_{y,i}^2\right) + Q_\theta e_{\theta,i}^2 + R_v v_i^2 + R_\omega \omega_i^2\right] \quad (6)$$

where Q_e , Q_θ , R_v and R_ω are weighting factors determining the priorities between the position error, orientation error and control actions. From the solution of the optimization problem, only the first calculated control is applied (v_0^*, ω_0^*), after which the process is repeated at the next discrete step. This procedure is known as a *receding horizon control* strategy.

under restrictions:

$$x_{k+1} = f(x_k, u_k), \quad u_k \in \mathcal{U}, \quad x_k \in \mathcal{X} \quad (7)$$

where $\mathcal{U}$ and $\mathcal{X}$ are the admissible sets of the control and states, respectively, and $f(\cdot)$ is the nonlinear model of the system.

2) Processing restrictions

One of the main advantages of MPC is the ability to incorporate physical and technological constraints into the optimization process. This includes:

- maximum linear speed: $v_{\min} \leq v \leq v_{\max}$;
- limited angular velocity range: $\omega_{\min} \leq \omega \leq \omega_{\max}$;
- acceleration restrictions: $\dot{v} \leq a_{\max}$;
- geometric constraints related to the turning radius.

Including these constraints ensures realistic robot behavior, prevents overloading of actuators, and guarantees control stability within physically permissible input signal values.

III. Experiments and Results

A. *Experiment*

We simulate mobile wheeled robot with foolowing parameters:

- wheelbase 0.3 [m];
- track 0.2 [m];
- height of center of mass 0.05 [m];
- mass 1.5 [kg];
- maximum speed 0.6 [m/s]

The following given trajectory is considered: The trajectory is formed by a cubic spline. In this experiment, the spline passes through six reference points with the following coordinates:

(x, y, z) = (0.00, 0.00, 0.00)
= (0.30, 0.50, 0.00)
= (1.00, 2.00, 0.00)
= (1.00, 3.00, 0.00)
= (2.00, 3.00, 0.00)
= (2.00, 4.00, 0.00)

Simulations are performed using the RE and PD methods [9], [10].

Fig. 8 and Fig. 9 show:

- the coordinate system in which the trajectories are displayed is scaled in meters
- a trajectory that is set is drawn along reference points, the number of which is selected by the user
- the set trajectory (dashed line); the trajectory the robot has traversed (blue line);
- a top view of the robot, with an arrow indicating the current direction of the resultant inertial vector and colored triangles changing color according to the level of danger of losing stability, relative to a given side of the robot;
- a colored bar, with shades of green, indicating the level of danger of losing stability, with light green corresponding to no danger, dark green indicating high danger, and red indicating that the robot has lost stability

B. *Results*

The results show that the trajectory deviation is less than 5 cm even with noise and a delay of 0.05 s. The MPC outperforms the PID controller in accuracy by about 40%.

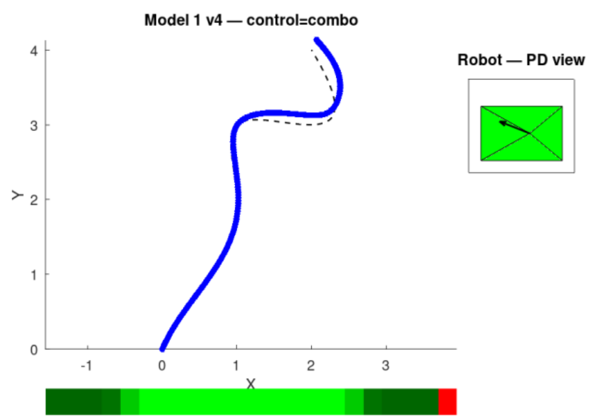

Fig. 9 Running a simulation using the RE method

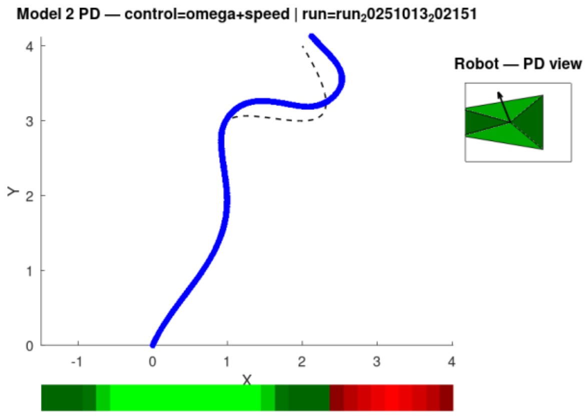

Fig. 10 Running a simulation using the PD method

IV. CONCLUSION

Predictive control is an effective method for tracking trajectories in non-holonomic mobile robots. By solving an optimization problem in real time, high accuracy and stability are achieved. Future research may include adaptive and deep-learning MPC approaches.

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