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Multifractal Wavelet-Based Analysis of Heart Rate Variability in Athletes Before and After Physical Exercise

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Abstract*—The present study presents an integrated approach for the analysis of heart rate variability (HRV), combining time-frequency wavelet analysis and multifractal WTMM analysis. The method allows for the assessment of dynamic changes in RR-interval series at rest and after physical exertion following a one-hour sports training session. For quantitative assessment of signal complexity, the Multifractal Hurst Complexity Index (MHCI) was used, which combines information from the multifractal spectrum and the Hurst exponent. The results show decreases in the spectrum width (Δα: 0.61→0.45) and MHCI (0.78→0.59), and an increase in asymmetry (2.30→2.72) after physical exertion, reflecting a reduction in the complexity and adaptability of the autonomic nervous system. Energy concentration at certain scales and a more uniform structure of the RR intervals were also observed. The proposed approach provides sensitive integral indicators for assessing the functional state of the cardiovascular system and has potential for application in monitoring stress and physical exertion.

***Keywords*—Continuous Wavelet Transform (CWT), Complexity sygnal, Hurst Rate Variability (HRV), multifractal signal, Wavelet Transform Modulus Maxima (WTMM).**

I. Introduction

Heart rate is not a strictly periodic process, but a complex dynamic system regulated by the interaction between the sympathetic and parasympathetic parts of the autonomic nervous system. Heart Rate Variability (HRV), derived from the sequence of heart rate intervals (RR intervals), reflects the cardiovascular system's ability to maintain homeostasis and respond to internal and external influences [1,2,13]. Therefore, HRV is considered an important non-invasive indicator of the functional state of the autonomic nervous system and of the body's adaptability to stressors [3].

Classical methods for HRV analysis, based on the time- and frequency-domain characteristics of RR intervals, provide valuable information about the level of variability and autonomic balance. Widely used indicators include SDNN, RMSSD, LF/HF ratio, and spectral power across different frequency ranges. Despite their informativeness, these methods are limited in their ability to describe the nonlinear, non-stationary, and multiscale nature of physiological signals. Biological systems, including the cardiovascular system, exhibit complex dynamics that linear statistical methods cannot fully describe.

Physiological responses to stress, whether mental or physical, result in changes in autonomic balance, typically characterized by increased sympathetic activity and decreased parasympathetic regulation. These changes are reflected in the dynamics of RR intervals and can be better analyzed by nonlinear methods capable of capturing scaling properties, singularities, and complex signal structure [7-9]. In recent years, multifractal analysis has emerged as an effective approach to study the complex dynamics of HRV, as it allows for the description of signals characterized by multiple scales and varying degrees of local regularity.

One of the most effective methods for multifractal analysis is the Wavelet Transform Modulus Maxima (WTMM), which combines wavelet transformation with modulus maxima analysis and enables singularity detection, determination of scaling exponents, and construction of the multifractal spectrum [4,10,11]. The complexity, variability, and adaptability of physiological signals can be assessed using the parameters of the multifractal spectrum. In addition to the width of the multifractal spectrum, an important characteristic of the multifractal signal structure is the asymmetry of the spectrum. It provides information about the distribution of local scaling exponents and the dominant type of fluctuations in the signal. A left- or right-asymmetric multifractal spectrum indicates a different participation of large and small fluctuations in the signal structure, which is related to the mechanisms of heart rate regulation. Therefore, the asymmetry of the multifractal spectrum can serve as an additional indicator of the cardiovascular system's structural organization and adaptability.

Despite growing interest in multifractal methods for HRV analysis, several significant limitations remain in the literature. Most studies have focused on individual multifractal parameters, such as spectral width or Hurst exponent, without using integral indicators to characterize the overall structural complexity of the signal [5,6]. In addition, relatively few studies combine multifractal WTMM analysis with time-frequency wavelet methods to study HRV during physical exertion, which complicates interpretation and limits practical application. Therefore, the development of integral multifractal indicators for the analysis of HRV under stressful conditions remains an open scientific problem.

In this regard, the present article proposes an integrated approach to the analysis of heart rate variability, based on multifractal WTMM analysis and the introduction of the Multifractal Hurst Complexity Index (MHCI), which combines information from the multifractal spectrum and the Hurst exponent into a single generalized quantitative indicator. The proposed approach enables a more robust assessment of changes in the structural complexity of RR intervals during physical exertion.

The study hypothesizes that physical exertion during training leads to significant changes in the multifractal characteristics of RR intervals, reflecting a decrease in the complexity of the cardiac signal and a change in the adaptability of the autonomic regulation.

The main objective of the present work is to investigate the influence of a stressful situation induced by physical exertion on the dynamics of heart rate, as measured by WTMM of RR intervals, and to use an index of multifractal complexity for the quantitative assessment of structural changes in the signal.

II. Guidelines For Manuscript Preparation

A. Data Ascusition

The study used heart rate recordings from 28 athletes aged 18-20 years, practicing wrestling, recorded in a state of rest and after physical exertion induced by a one-hour training session. Electrocardiographic (ECG) signals were recorded with a Holter device, with recordings made for 15 minutes before the start of the training and 15 minutes immediately after its completion. The RR intervals, representing the time between consecutive R-peaks, were extracted from the ECG signals and used for subsequent analysis of heart rate variability. The study was conducted with approval from the Ethics Committee, and all participants signed informed consent to participate in the study. All participants were clinically healthy and did not have cardiovascular diseases.

B. Continuous Wavelet Transform

To analyse the non-stationary properties of the RR interval series, a continuous wavelet transform (Continuous Wavelet Transform , CWT) was used, which provides both time and frequency representations of the signal. This method is particularly suitable for analysing physiological signals characterised by variable-frequency components and non-stationary behaviour. In the present study, the Morlet wavelet function was used for its good time-frequency localisation and its ability to analyse non-stationary biomedical signals effectively.

The continuous wavelet transform is defined as:

$$W(a,b) = \frac{1}{\sqrt{a}} \int x(t)\psi\left(\frac{t-b}{a}\right) dt \tag{1}$$

where

- a –scale parameter;
- b – time parameter;
- ψ – wavelet function.

C. Wavelet Transform Modulus Maxima (WTMM)

The multifractal analysis was performed using the WTMM method. The method is based on locating the wavelet transform's modular maxima at different scales and tracking their evolution [4]. By analysing the scaling behaviour of these maxima, the signal's singularities and multifractal properties can be determined.

The partition function is calculated with the following equation:

$$Z(q,a) = \sum |W(a,b)|^q \tag{2}$$

The dependence between Z(q, a) and the scale a determines the scaling function τ(q), from which the multifractal spectrum f(α) is obtained through Legendre transformation.

D. Multifractal Spectrum

The multifractal spectrum f(α) describes the distribution of singularities in the signal and provides information about its complexity and scaling properties [5,12]. The main parameters of the spectrum are:

- spectrum width Δα;
- position of maximum α_0;
- spectrum asymmetry.

The width of the spectrum, which characterises the degree of multifractality and complexity of the signal, is determined by the following equation:

$$\Delta\alpha = \alpha_{max} - \alpha_{min} \tag{3}$$

E. Asymmetry of Multifractal Spectrum

The asymmetry of the multifractal spectrum characterises the shape of the spectrum and the distribution of fluctuations in the signal. It indicates whether small or large fluctuations dominate the multifractal structure and can serve as an indicator of the signal's structural organisation and the dynamics of autonomic regulation. Positive asymmetry is usually associated with the dominance of small fluctuations and a more stable signal structure. In contrast, negative asymmetry indicates a greater influence of large fluctuations and more uneven dynamics. Changes in the asymmetry of the multifractal spectrum may reflect changes in the regulatory mechanisms of the autonomic nervous system and the body's adaptive responses to stress or physical exertion.

F. Multifractal Hurst Complexity Index (MHCI)

The Multifractal Hurst Complexity Index, based on the multifractal spectrum and the Hurst exponent, was used to quantify signal complexity. This index is an integral indicator of the signal's structural complexity, correlation properties, and multifractality. By combining information on the long-term dependence described by the Hurst exponent with the characteristics of the multifractal spectrum, the index enables a more complete assessment of the signal's nonlinear dynamics. Higher MHCI values correspond to greater complexity, better structural organisation, and greater adaptability of the physiological system, while lower values indicate reduced complexity and a more limited regulatory capacity. Therefore, the index can be used as an integral indicator for assessing the functional state of the autonomic nervous system and the body's adaptive responses in different physiological states, such as rest, stress, and physical exertion. An advantage of MHCI is that it combines information about the scaling behaviour, correlation structure, and multifractal properties of the signal into a single generalised parameter, facilitating comparative analysis across different physiological states and subject groups.

G. *Statistical Analysis*

A t-test was used to compare the studied parameters between the rest and physical exertion states, with $p < 0.05$ as the criterion for statistical significance. All results are presented as mean ± standard deviation (mean ± SD).

III. Results

This section presents the main results of the HRV analysis in the studied athletes at rest and after physical exertion. The aim is to illustrate the changes in the dynamics of RR intervals and signal structure related to the adaptation of the autonomic nervous system to physical exertion.

Figure 1 shows the RR interval series at rest and after exercise. Clearly pronounced changes in the variability and structure of the signal are observed after exercise, which is an indicator of changes in the autonomic regulation of cardiac activity.

Figure 2 illustrates the wavelet scalograms of the RR intervals for the two conditions. The scalograms show a different distribution of energy across scales, indicating a change in the frequency structure of the signal after physical exertion.

Figure 3 presents the dependence of the Partition function Z(q,a) on the scale for different values of q. The obtained linear dependences on the logarithmic scale confirm the presence of fractal behaviour in the RR interval series.

Figure 4 shows the multifractal spectrum f(α) for the resting state and after physical exertion. Differences in the width and shape of the spectrum are observed, indicating changes in the complexity and fractal structure of the signal after physical exertion.

Table I summarizes the values of the main multifractal parameters – spectrum width (Δα), spectrum asymmetry and multifractal complexity index (MHCI) at rest and after physical exertion.

TABLE I. Statistical parameters of variables at Rest and Stress

Parameter	**Rest** mean±std	**Stress** mean±std	**p-value** mean±std
Multifractal spectrum	0.61±0.10	0.45±0.09	<0.0001
MHCI	0.78±0.09	0.59±0.07	<0.0001
Asymmetry	2.30±0.21	2.72±0.35	<0.0001

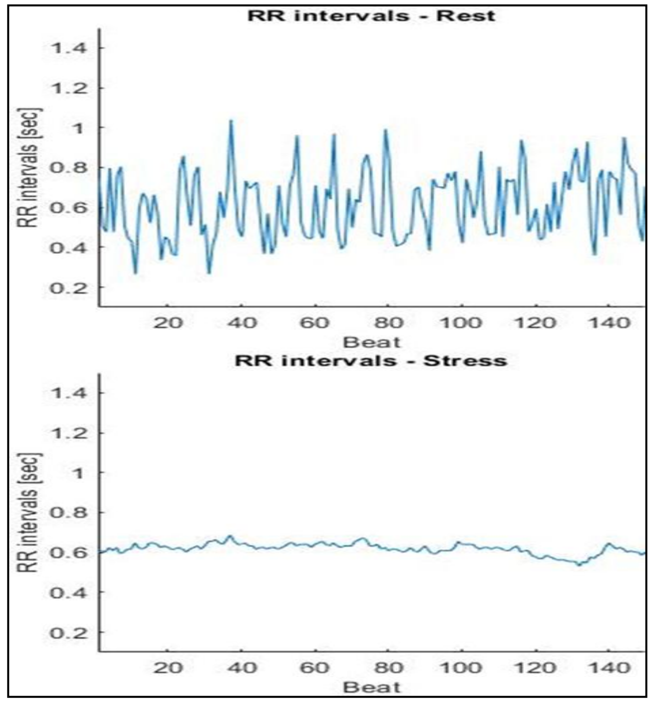

Fig 1. RR interval time series for rest and post-training conditions

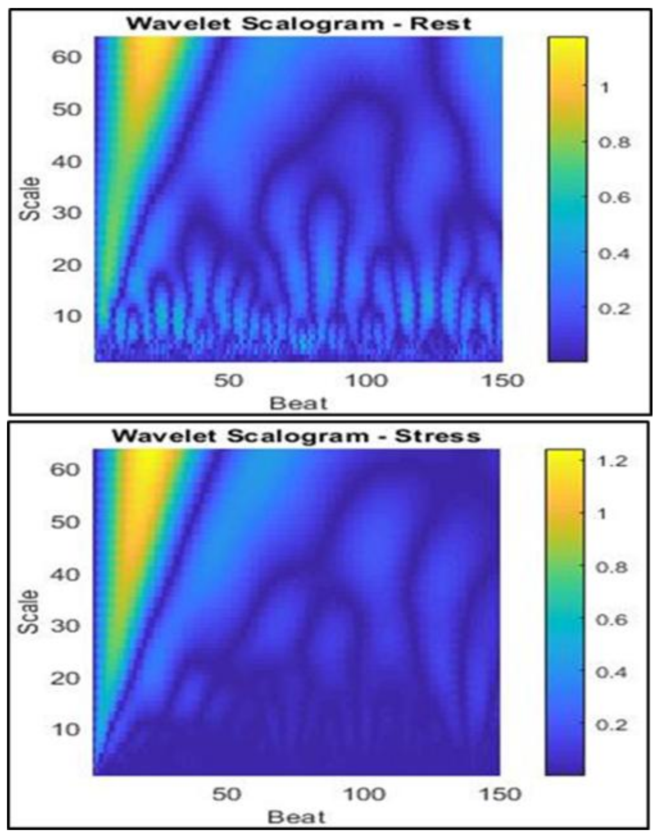

Fig. 2. Wavelet scalogram of RR intervals

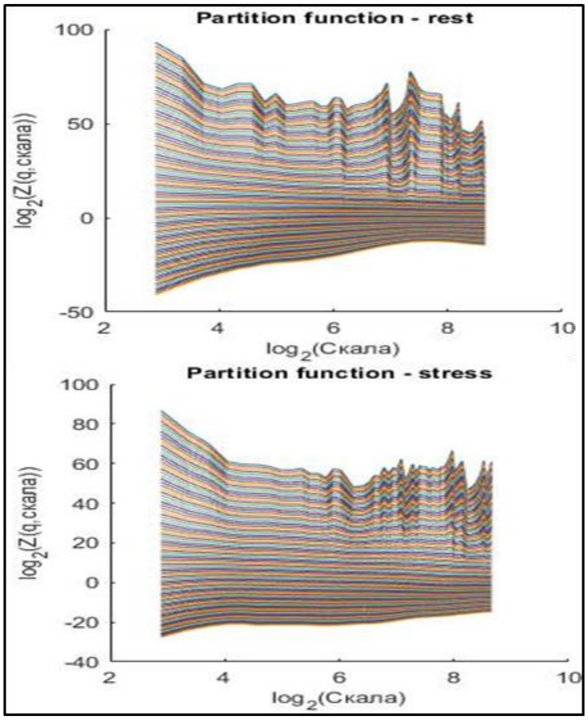

Fig. 3. Partition function of RR intervals

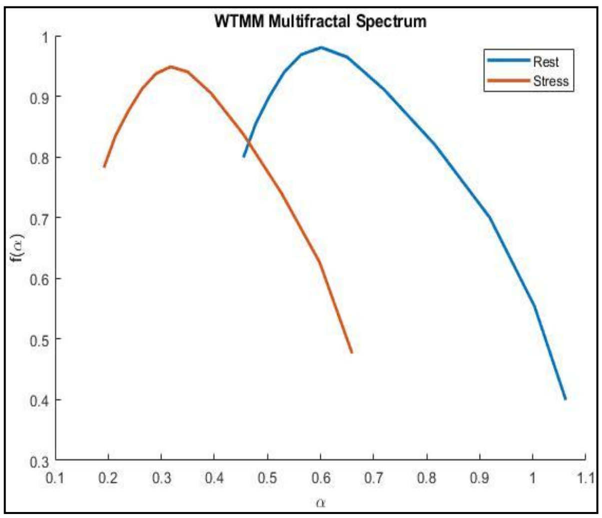

Fig. 4. Multifractal spectrum f(α) of RR intervals

IV. DISCUSSION

In the present study, multifractal analysis of RR intervals based on the WTMM method revealed significant alterations in the structural complexity of cardiac rhythm after one hour of exercise. The results summarized in Table 1 demonstrate significant post-exercise changes in heart rate. Specifically, a decrease in the spectrum width ($\Delta\alpha$) and the multifractal heart rate complexity index (MHCI) was observed, accompanied by an increase in the spectrum asymmetry index. These changes suggest a modification in autonomic nervous system balance and altered cardiovascular regulation following physical load.

The decrease in the width of the spectrum ($\Delta\alpha$) reflects less variability of the RR intervals on different time scales, which is an indicator of a reduction in the complexity of cardiac regulation. Changes in asymmetry indicate that the dominant fluctuations change: at rest the spectrum is slightly left-sided, which reflects a greater participation of large fluctuations; after exercise the spectrum becomes more symmetrical, indicating a decrease in the participation of large and sharp fluctuations. The decrease in the MHCI index is of particular importance, since this index is related to the Hurst parameter, which characterizes the long-term correlation and fractal properties of heart rate variability. Higher values of the Hurst parameter are associated with greater complexity and better adaptability of the cardiovascular system, while values close to 0.5 indicate a more random nature of the signal and reduced regulatory capacity of the autonomic nervous system. In the present study, a decrease in MHCI from 0.78 at rest to 0.59 after exercise was observed, which can be interpreted as a decrease in the fractal complexity of the heart rhythm and a decrease in the adaptability of the body as a result of physical stress and fatigue.

The increase in asymmetry also indicates an increased load on regulatory mechanisms and a change in autonomic balance in the direction of increased sympathetic activity and decreased parasympathetic activity after physical exertion.

The analysis of the partition function shows a clear difference between the state of rest and the state after physical exertion. In the state of rest, a greater separation of the curves is observed for different values of the parameter q, which indicates a strong dependence on q and the presence of a well-expressed multifractal behaviour of the signal. This indicates that the dynamics of the RR intervals in the state of rest is characterized by a complex multifractal structure. In the state of stress, the partition function curves are closer to each other and the dependence on the parameter q is less pronounced, which indicates a reduction in the multifractal properties and a transition to a more monofractal behaviour of the signal. This means that during physical exertion, the dynamics of the heart rate becomes simpler and less complex. These results are consistent with the obtained reduction in the multifractal width $\Delta\alpha$ and the MHCI index and confirm the hypothesis of a reduction in physiological complexity during stress and physical exertion.

This combination of changes in the studied parameters indicates that physical exertion causes a reduction in the complexity of the heart rhythm and reduced adaptability, accompanied by a concentration of energy at certain scales, also observed in the Wavelet scalogram (Figure 2).

Therefore, physical exertion leads to a decrease in heart rate variability, a loss of heart rhythm complexity, and reduced adaptability of the autonomic nervous system.

The results are consistent with the concept of “loss of complexity” described by the authors [13], according to which physiological stress and exercise lead to a decrease in HRV complexity. The reduction in $\Delta\alpha$, asymmetry, and

MHCI is consistent with previous observations that stressful conditions reduce the multifractal properties of RR intervals and the adaptability of the autonomic nervous system. Similar effects have been reported for both exercise and cognitive or psycho-emotional stress.

The reduction in multifractal complexity observed after exercise likely reflects a shift toward sympathetic dominance and reduced parasympathetic modulation. This suggests that the cardiovascular system adapts to physical load by optimizing regulatory mechanisms, leading to more uniform and less complex RR interval dynamics. Additionally, the observed changes in spectrum asymmetry indicate a redistribution of dominant fluctuation scales, which may represent a specific adaptive response of the cardiovascular control system to physiological stress.

The integrated WTMM approach, combining multifractal spectrum analysis and MHCI, allows for both quantitative and qualitative description of changes in the structural complexity of RR intervals. The asymmetry analysis provides additional information about the nature of fluctuations and dynamic organization of the signal, which would be difficult to capture using classical time and frequency indices. Thus, this approach can be used for monitoring physiological stress and assessing the adaptability of the cardiovascular system in sports and clinical settings.

V. STUDY LIMITATION

The current study has several limitations. The number of participants was limited, which may affect the statistical power of the analysis. In addition, the focus was on a one-hour exercise session and the immediate effects after exercise; long-term changes and the influence of different types of physical activity should be investigated in future studies. The interpretation of integral multifractal indices, including MHCI, also requires further validation in different populations.

Despite these limitations, the present study demonstrates that multifractal WTMM analysis and MHCI are effective tools for assessing the functional state of the autonomic nervous system and heart rate dynamics during exercise. The results obtained confirm the concept of complexity reduction under stressful conditions and highlight the potential of integral multifractal metrics as sensitive markers of adaptability and physiological reactivity.

VI. CONCLUSION

The present study demonstrates that multifractal WTMM analysis combined with the multifractal complexity index provides sensitive and integral indicators of the structural complexity and adaptability of the cardiovascular system. The results obtained show that one hour of physical exercise leads to a significant decrease in multifractal width ($\Delta\alpha$), a change in spectral asymmetry, and a decrease in MHCI, reflecting a reduction in the complexity and adaptability of RR intervals under stress. These findings support the concept of a reduction in physiological complexity under stress conditions and highlight the potential of integral multifractal indicators as sensitive markers of the functional state of the autonomic nervous system.

The proposed multifractal approach expands the possibilities for quantitative analysis of heart rate variability and can be used for stress monitoring, assessment of recovery and adaptation to sports loads, as well as in clinical practice for assessing the functional state of the autonomic nervous system. This defines multifractal indicators and the MHCI index as promising quantitative markers for assessing physiological stress and the adaptive capacity of the body.

In summary, multifractal analysis of HRV can be considered as a reliable tool for quantitatively characterizing complex physiological signals and for assessing the adaptability of the organism to stressful influences.

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